Heuristics of Approximation of Subgraph Centrality Metric for Dynamic Graphs

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Outline

Importance

Addition of new edge in the graph

Approximation of subgraph centrality

- Idea
- Algorithm

Results

Future plans

Importance

Graph algorithms

- Bioinformatics
- Social network analysis
- Business analytics
- Knowledge discovery
- City planning
- *and others…*

Importance

How to distinguish nodes in graph from each other?

- Use centrality metrics
  - Degree
  - Closeness
  - Betweenness
  - etc.

Centrality metric with best discriminative power

- Subgraph centrality
- E. Estrada, J.A. Rodrigues-Velazques “Subgraph centrality in complex networks” // Physical Review E 71 (5), 056103

Subgraph centrality

Characterize the participation of each node in all subgraph in a network

Number of closed walks starting and ending at the node

$C_s = diag\ e^A$, where

$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots + \frac{A^k}{k!} + \cdots,$

A – adjacency matrix

Cubic complexity to compute!

Addition of new edge in the graph

What if?

- We have large graph (thousands of nodes or more)
- We have computed exact values of subgraph centrality metric
- But what we should do if new edge appears in this graph?
  - Compute all exact values again?
  - Or somehow try to approximate / assess new values of subgraph centrality after perturbation in graph?

Main goal

Try to approximate values of subgraph centrality metric in dynamic graphs

We use different graphs for this purpose

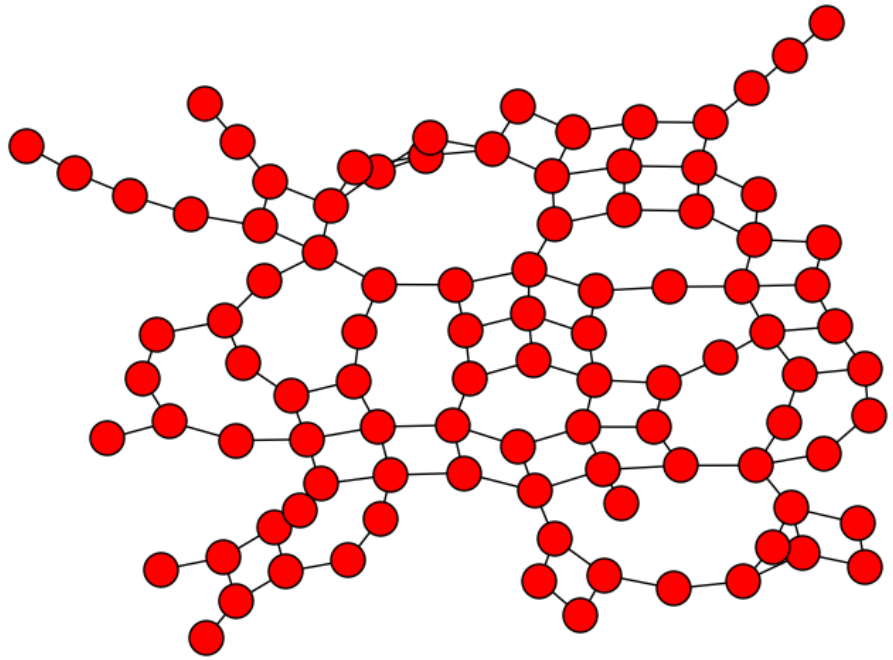
- Erdős-Rényi graphs
  - “Low” diameter, random structure
- Models of road networks
  - “High” diameter, grid structure

Approximation of subgraph centrality

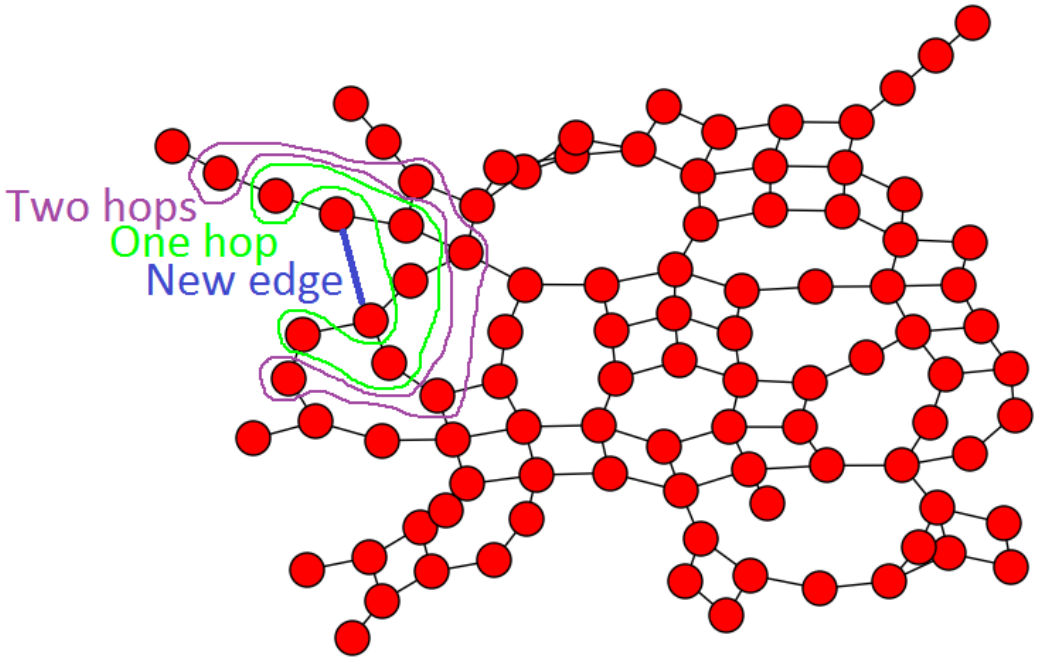
Idea

- Lets see, what happens with exact values of subgraph centrality metric of vertices , between which we input an edge, and its nearest neighbors
- Maybe all changes are *local*?

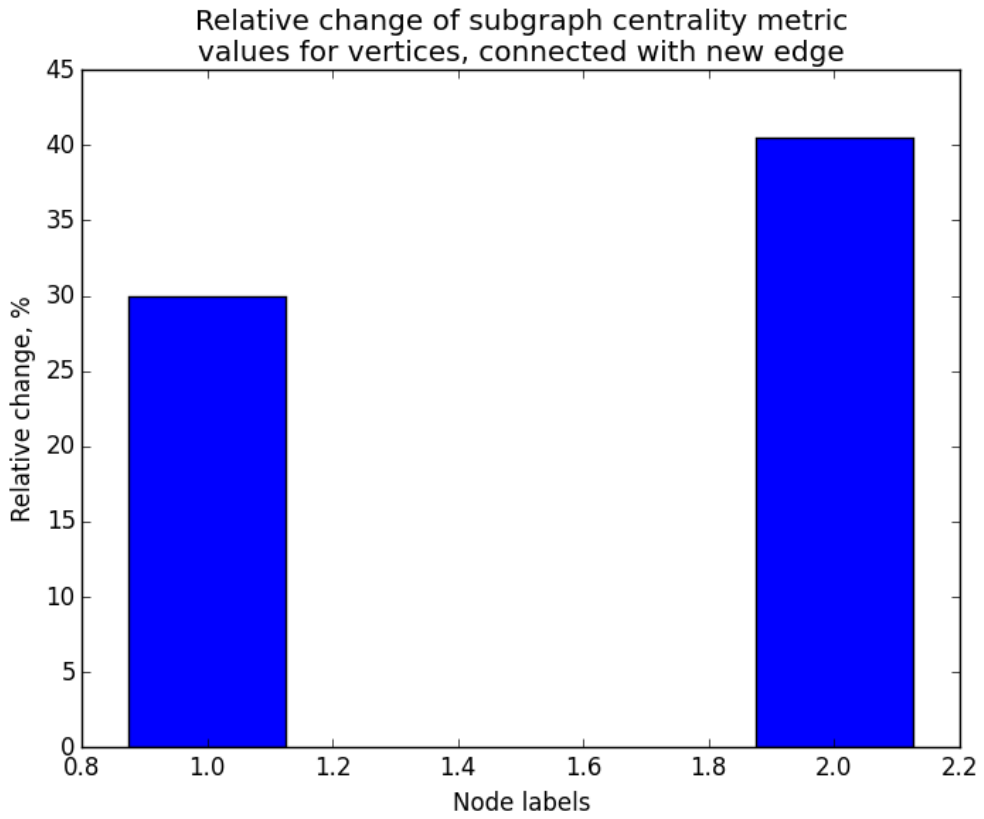
Approximation of subgraph centrality



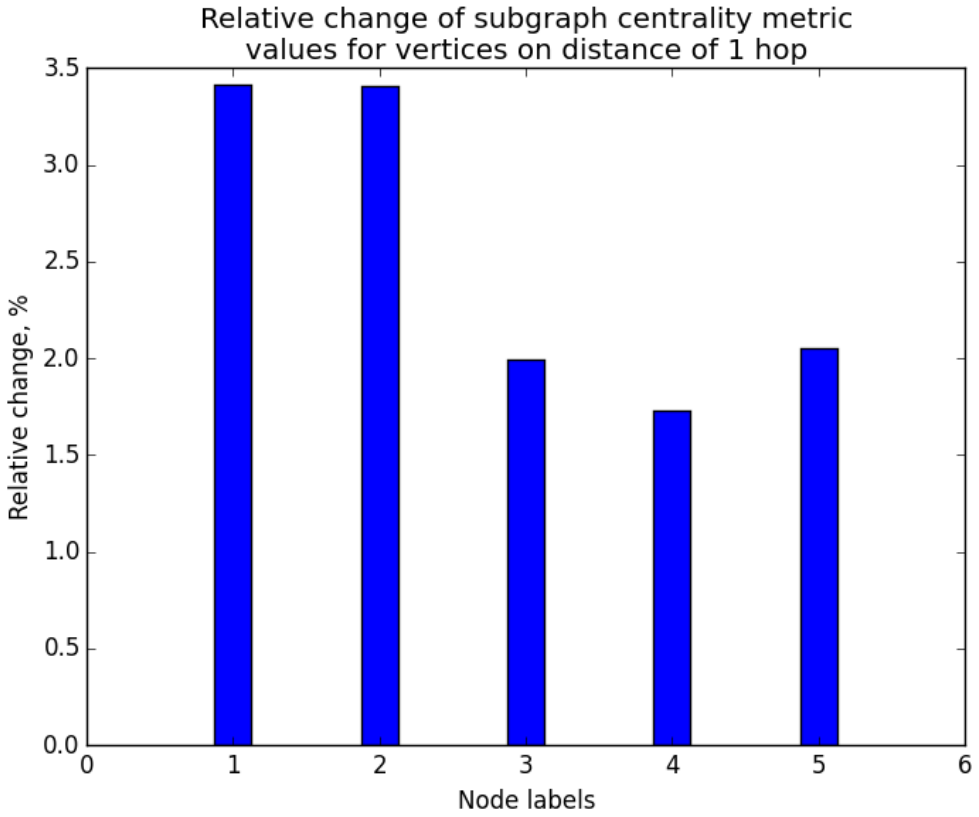
Approximation of subgraph centrality



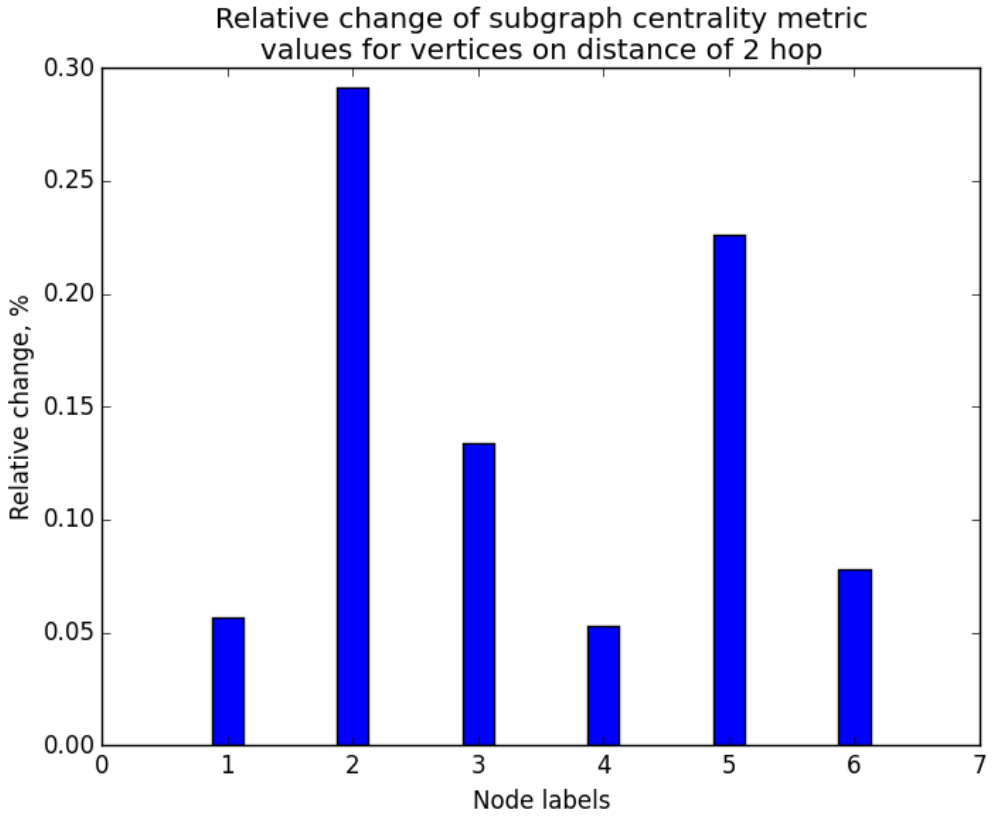
Approximation of subgraph centrality



Approximation of subgraph centrality



Approximation of subgraph centrality



Approximation of subgraph centrality

Algorithm

```
S1 ← extract_subgraph(G1, vertex_list)
S2 ← extract_subgraph(G2, vertex_list)
S1_centr ← compute_centrality(S1)
S2_centr ← compute_centrality(S2)
for(i=0; i<len(G1); i++)
   G2_centr[i] = G1_centr[i]
for(i=0; i<len(vertex_list); i++)
   pos ← vertex_list[i]
   alpha[i] ← 1+(S2_centr[i]-S1_centr[i])/S1_centr[i]
   G2_centr[pos] ← alpha[i] x G1_centr[pos]
```

Graphs for testing

Types of graphs (undirected)

- Erdős-Rényi graphs
- Models of road networks

Software

- NetworkX 1.9.1-1

Hardware

- For Erdős-Rényi graphs
  - Intel Core i7-2630QM
  - 4 GB RAM
- For models of road networks
  - Intel Pentium 3556U
  - 4 GB RAM

Road networks generation

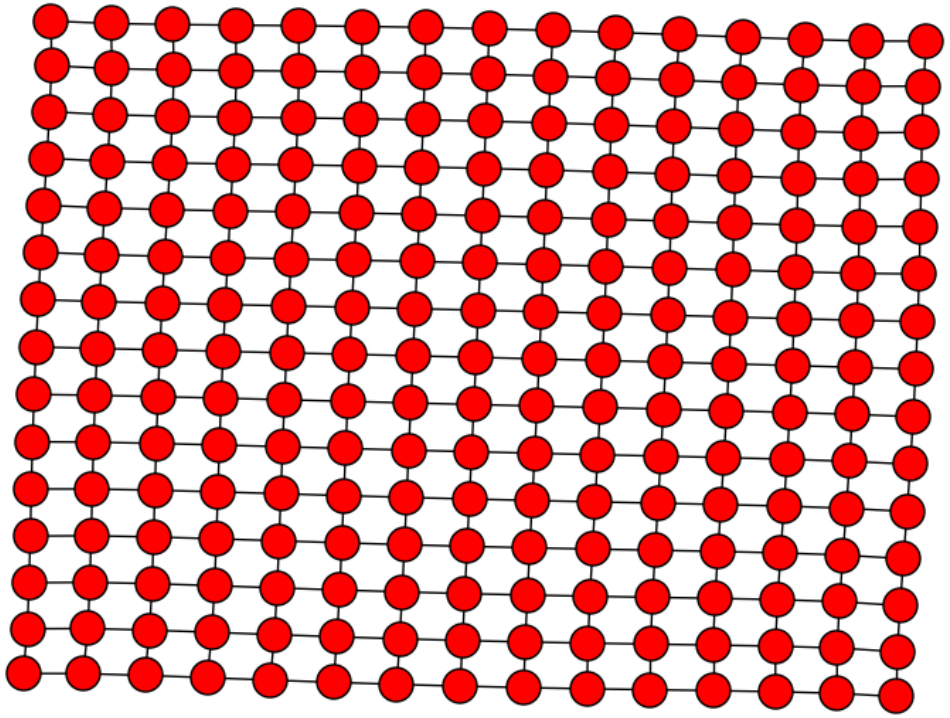
First step

- Make a grid

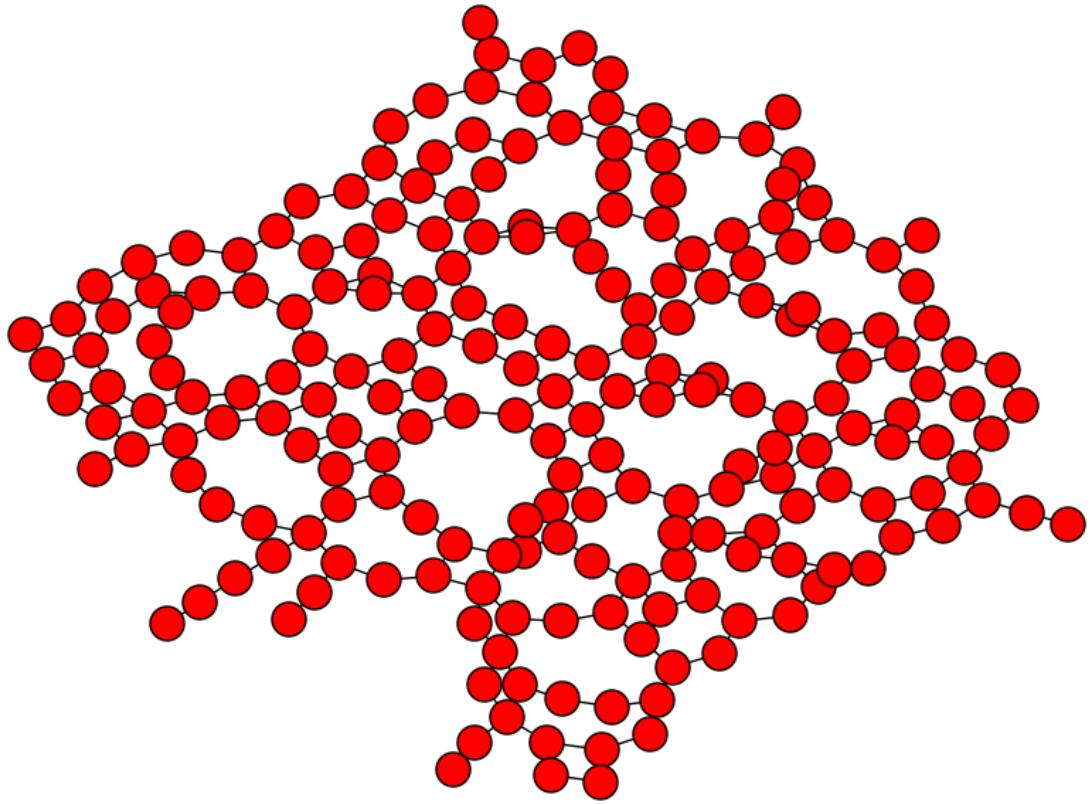
Second step

- Sparsify grid
- Keep all deleted edges!

Road networks generation (first step)



Road networks generation (second step)



Graphs for testing

ERDŐS-RÉNYI

ER_10000_00035
- nodes = 9670, edges = 17393, diameter = 16

ER_10000_00030
- nodes = 9429, edges = 14940, diameter = 19

ER_10000_00025
- nodes = 8893, edges = 12106, diameter = 24

ER_10000_00020
- nodes = 8067, edges = 9705, diameter = 33

ROAD NETWORKS

RN_1600
- nodes = 1547, edges = 2011, diameter = 83

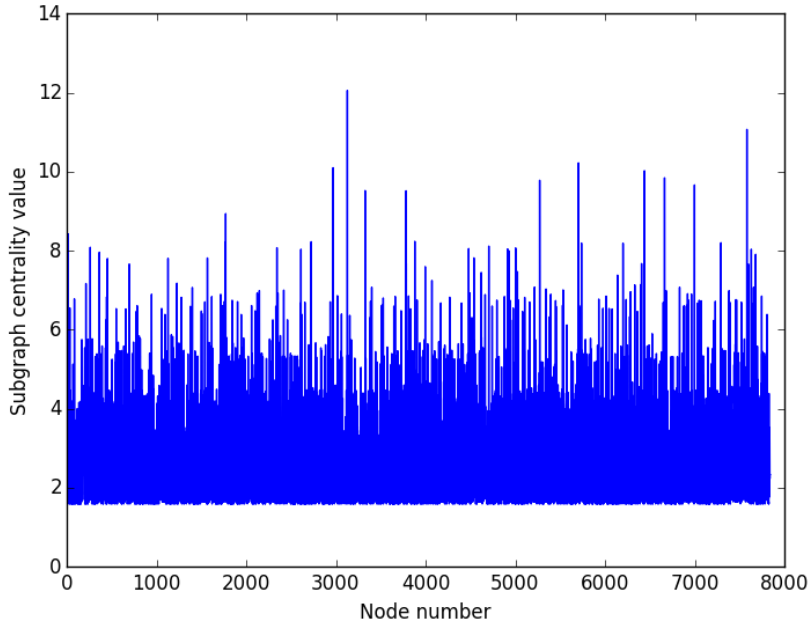
RN_2500
- nodes = 2400, edges = 3141, diameter = 101

RN_3600
- nodes = 3505, edges = 4578, diameter = 122

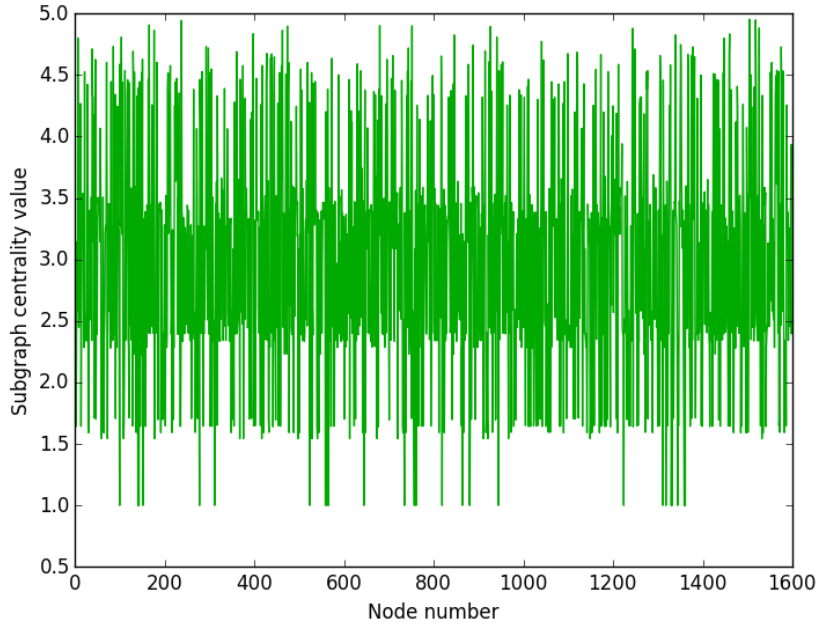
RN_4900
- nodes = 4747, edges = 6234, diameter = 139

Results (exact values)

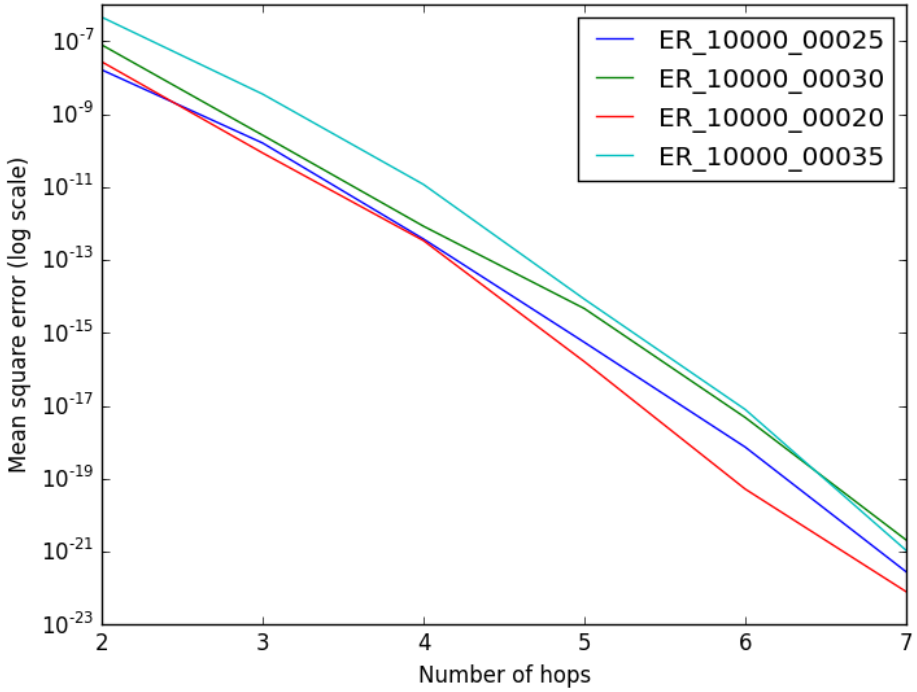
ER_10000_00020

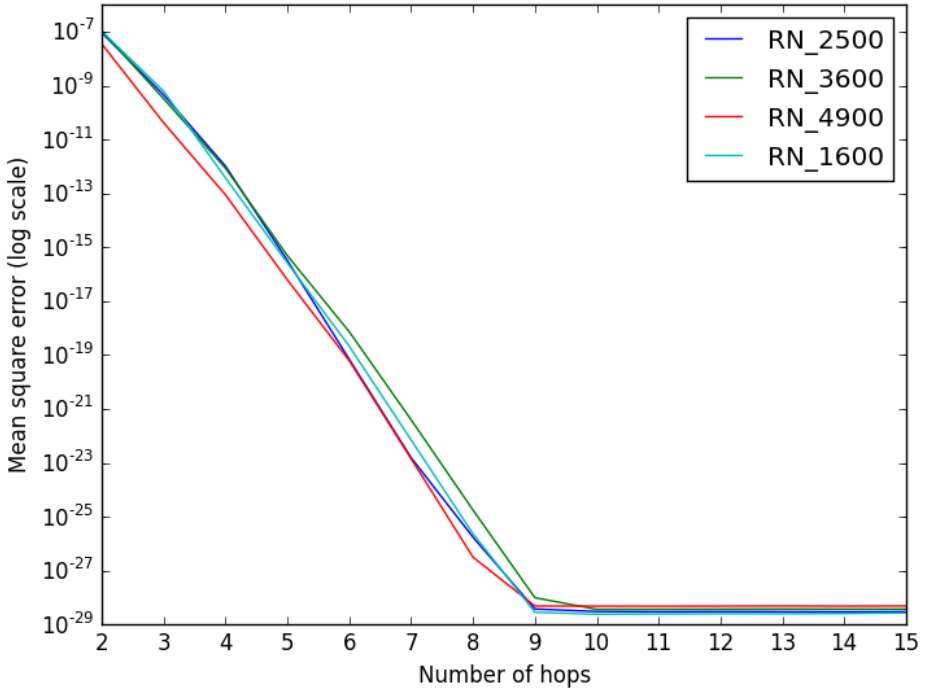


RN_1600

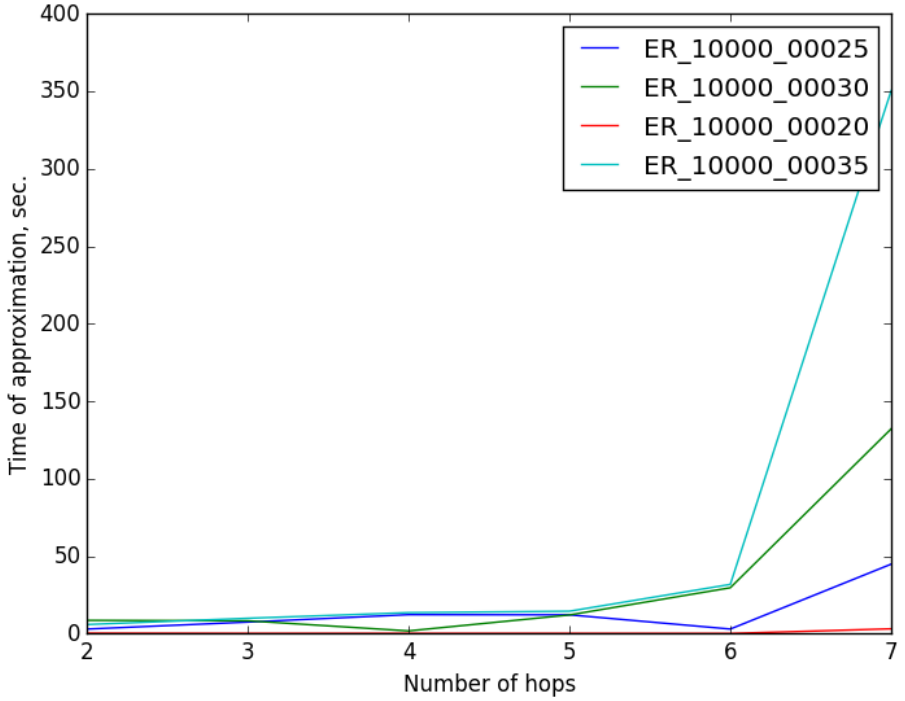


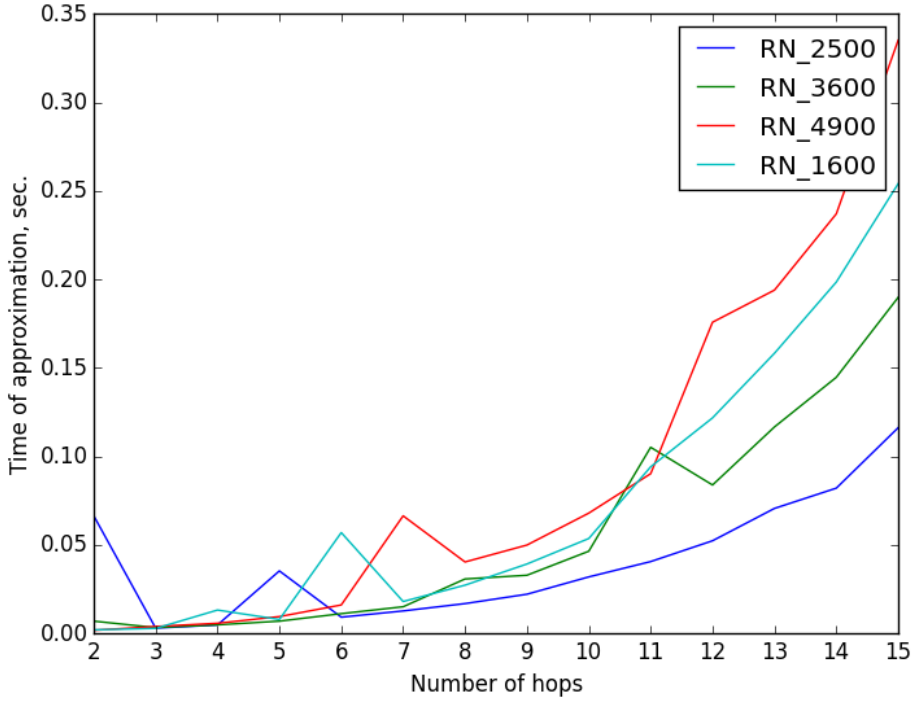
Results (mean square error)



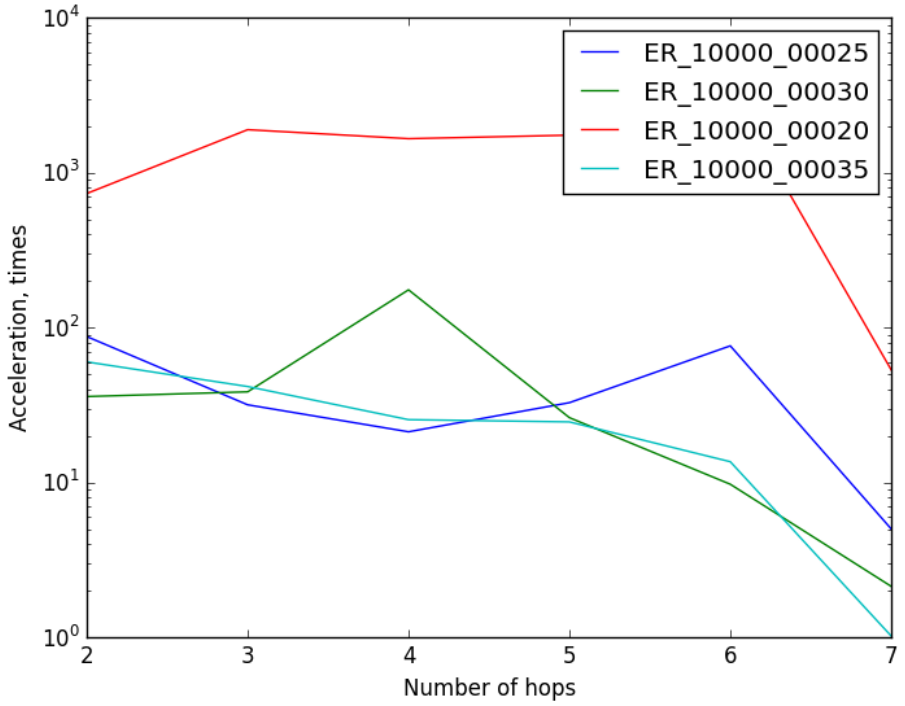


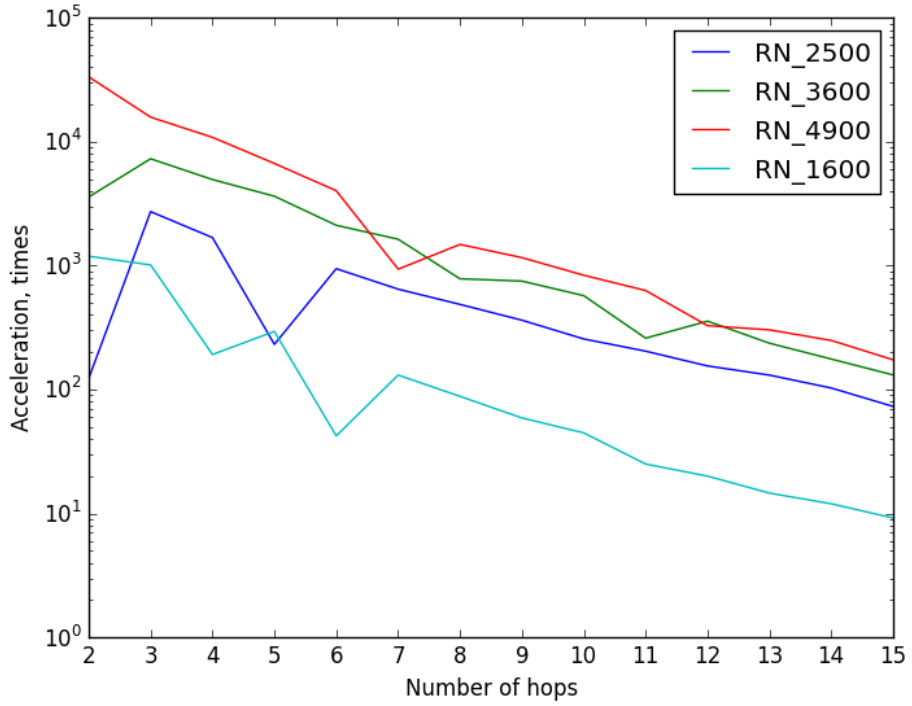
Results (time of approximation)



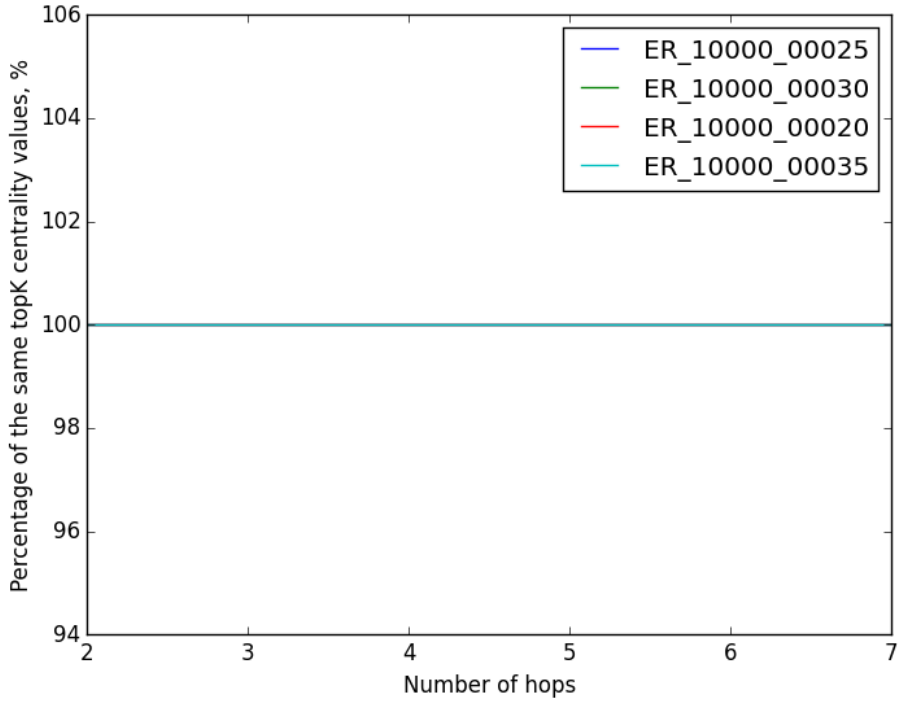


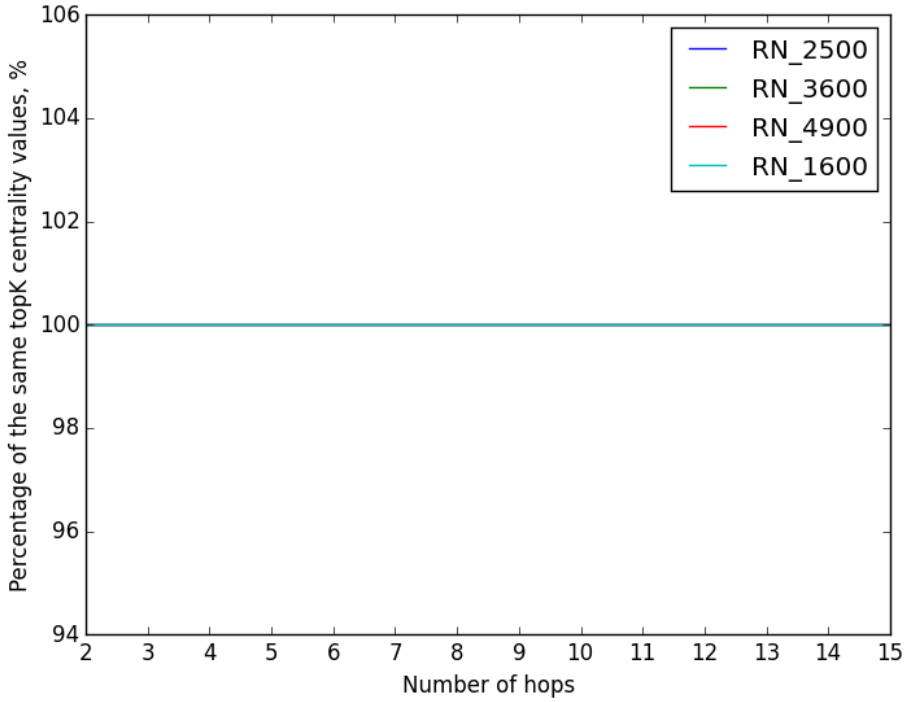
Results (acceleration)





Results (topK)





Future plans

Approximation of centrality metrics in

- Weighted graphs
- Directed graphs
- Graphs, obtained from real world tasks

Try to analyze algorithm behavior and convergence when adding many edges

Try to tune algorithm for small-world graphs

Questions?